KINEMATIC ANALYSIS OF MONO-MOBILE MECHANICAL SYSTEM ACTUATED BY MOTOR TETRAD RRR-TA-RRR

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Abstract. The mono – mobile mechanical systems are composed of an active modular group and one or more passive modular groups (dyad, triad etc.). To calculate the kinematic parameters of active modular groups there can be used methods such as the contour method, matrix method, grapho-analytical method, graphics method etc. In the case of solving the positions of the complex mechanisms as a whole, systems with a large number of equations, usually non-linear, are obtained. For solving these systems of equations the function matrix of the system is required, the matrix, which elements are obtained in a numerical or analytical way. The number of equations is very large, and the size of the functional matrix is very large, which leads to a large number of operations to obtain the inverse matrix. This paper presents a procedure for the kinematic analysis of an active modular group motor tetrad RRR-Ta-RRR type. To calculate the kinematic parameters (positions, angular velocities and accelerations) the contour method is used. The active modular group RRR-Ta-RRR has two independent kinematic contours, making it particularly effective in operating various mobile mechanical systems. The kinematic performances of the mechanisms, in which this modular group is found, are very high, the numerical solution of the system of non-linear equations of positions is simple, the convergence is rapid and the calculation precision desired can be controlled very easily. As examples of the use of active modular group RRR-Ta-RRR or other active modular groups (motor dyad, motor triad) can be mentioned: actuation of various lifting systems (actuating mechanism of a crank with articulated bars, actuating mechanism of a crank with articulated bars and gears), actuation of different folding tables, actuation of the gripping mechanisms used in industrial robots, etc.

Keywords: active modular group, mechanical system, kinematic analysis.

Introduction

The main objective of this paper is to present the procedure for calculating the kinematic parameters of an active modular group. The active modular group is used to activate the mechanical systems. The equations for kinematic analysis of the active modular group RRR-Ta-RRR [1-11] are written using the contour method [8]. As an example of calculation a lifting mechanism is analyzed, which has in its structure a motor tetrad and two RRR types. For kinematic analysis of the mechanism we used calculation procedures written by the authors in the MATLAB syntax. To create a clearer image how the mechanism works, the authors of the paper have also compiled animation computational procedures, and this animation has led to the theoretical data being confirmed.

Materials and methods

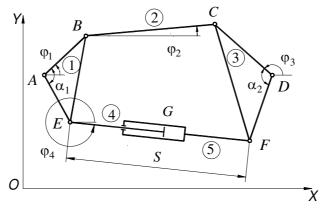Figure 1 presents the active modular group named motor tetrad [5-9;12].

Fig.1. **Kinematic scheme of motor tetrad**

The active modular group RRR-T$_a$-RRR, formed by five mobile elements and seven lower joints, has one degree of mobility.

To calculate the kinematic parameters of this active modular group the following are known: the length of all kinematic elements, positions, velocities and accelerations of points A, D, G and the approximate coordinates of joints B, C, E and F. (Table 1) [5-9;12].

Table 1

Input data of RRR-T$_a$-RRR active modular group

Description	Parameters
Length of kinematic elements	$AB, AE, BE, BC, CD, DF, CF$
Coordinates of points A and D	XA, YA, XD, YD
Projections of linear velocities of points A and D	$\dot{X}A, \dot{Y}A, \dot{X}D, \dot{Y}D$
Projections of linear accelerations of points A and D	$\ddot{X}A, \ddot{Y}A, \ddot{X}D, \ddot{Y}D$
Variable parameter in the active joint G	S
Linear relative velocity of the active joint G	$\dot{S}$
Linear relative acceleration of the active joint G	$\ddot{S}$
Approximate coordinates of the internal joints B, C, E and F	$XB, YB, XC, YC, XE, YE, XF, YF$

In Table 2, the dependent kinematic parameters that will be determined are presented [5-9;12].

Table 2

Output data of RRR-T$_a$-RRR active modular group

Description	Parameters
Angles between the axis OX and the vectors $\overline{AB}, \overline{BC}, \overline{DC}, \overline{EF}$	$\varphi_1, \varphi_2, \varphi_3, \varphi_4$
Angular velocities of the links 1, 2, 3 and 4(5)	$\dot{\varphi}_1, \dot{\varphi}_2, \dot{\varphi}_3, \dot{\varphi}_4$
Angular accelerations of the links 1, 2, 3 and 4(5)	$\ddot{\varphi}_1, \ddot{\varphi}_2, \ddot{\varphi}_3, \ddot{\varphi}_4$

1. Analysis of the positions of active modular group RRR-T$_a$-RRR

The vector equations [5, 6, 7, 8, 9, 10]:

$$\overline{OA} + \overline{AB} + \overline{BC} = \overline{OD} + \overline{DC} ; \quad \overline{OA} + \overline{AE} + \overline{EF} = \overline{OD} + \overline{DF} , \qquad (1)$$

are projected on the axis of the coordinate system and the following four non – linear equations are obtained:

$$\begin{cases} AB\cos\varphi_1 + BC\cos\varphi_2 - CD\cos\varphi_3 + XA - XD = 0 \\ AB\sin\varphi_1 + BC\sin\varphi_2 + CD\sin\varphi_3 + YA - YD = 0 \\ AE\cos(\varphi_1 - \alpha_1) + S\cos\varphi_4 - DF\cos(\varphi_3 + \alpha_2) + XA - XD = 0 \\ AE\sin(\varphi_1 - \alpha_1) + S\sin\varphi_4 - DF\sin(\varphi_3 + \alpha_2) + YA - YD = 0 \end{cases} \qquad (2)$$

The unknowns of the system of equations (2) are φ_1, φ_2, φ_3 and φ_4.

To solve the non–linear system of equations (2) the iterative Newton–Raphson method is used starting from the given initial solution.

The functions of the non – linear system of equations (2) are:

$$f_1(\varphi_1,\varphi_2,\varphi_3,\varphi_4)=AB\cos\varphi_1+BC\cos\varphi_2-CD\cos\varphi_3+XA-XD$$

$$f_2(\varphi_1,\varphi_2,\varphi_3,\varphi_4)=AB\sin\varphi_1+BC\sin\varphi_2-CD\sin\varphi_3+YA-YD$$

$$f_3(\varphi_1,\varphi_2,\varphi_3,\varphi_4)=AE\cos(\varphi_1-\alpha_1)+S\cos\varphi_4-DF\cos(\varphi_3+\alpha_2)+XA-XD$$

$$f_4(\varphi_1,\varphi_2,\varphi_3,\varphi_4)=AE\sin(\varphi_1-\alpha_1)+S\sin\varphi_4-DF\sin(\varphi_3+\alpha_2)+YA-YD$$

The system solution at the $(i + 1)$ iteration has the form:

$$\begin{Vmatrix}\varphi_1\\\varphi_2\\\varphi_3\\\varphi_4\end{Vmatrix}^{i+1}=\begin{Vmatrix}\varphi_1\\\varphi_2\\\varphi_3\\\varphi_4\end{Vmatrix}^{i}-W^{-1}(\varphi_1^i,\varphi_2^i,\varphi_3^i,\varphi_4^i)\begin{Vmatrix}f_1(\varphi_1^i,\varphi_2^i,\varphi_3^i,\varphi_4^i)\\f_2(\varphi_1^i,\varphi_2^i,\varphi_3^i,\varphi_4^i)\\f_3(\varphi_1^i,\varphi_2^i,\varphi_3^i,\varphi_4^i)\\f_4(\varphi_1^i,\varphi_2^i,\varphi_3^i,\varphi_4^i)\end{Vmatrix},\tag{3}$$

where

$$W=\begin{Vmatrix}-AB\sin\varphi_1 & -BC\sin\varphi_2 & CD\sin\varphi_3 & 0\\AB\cos\varphi_1 & BC\cos\varphi_2 & CD\cos\varphi_3 & 0\\-AE\sin(\varphi_1-\alpha_1) & 0 & DF\sin(\varphi_3+\alpha_2) & -S\sin\varphi_4\\AE\cos(\varphi_1-\alpha_1) & 0 & -DF\cos(\varphi_3+\alpha_2) & S\cos\varphi_4\end{Vmatrix},\tag{4}$$

represents the matrix of the partial derivatives of the system functions in relation to the parameters φ_1, φ_2, φ_3 and φ_4.

The initial solution is:

$$\phi_1^{(0)}=\operatorname{atan}2(YB-YA,XB-XA);\ \phi_3^{(0)}=\operatorname{atan}2(YC-YD,XC-XD)$$

$$\phi_2^{(0)}=\operatorname{atan}2(YC-YB,XC-XB);\ \phi_4^{(0)}=\operatorname{atan}2(YF-YE,XF-XE),\tag{5}$$

where XB, YB, XC, YC, XE, YE, XF, YF are the approximate coordinates of internal joints B, C, E and F.

The iterative process stops, when the difference of two consequently calculated solutions is smaller than ε (imposed value) meaning:

$$\|\phi\|^{(i+1)}-\|\phi\|^{(i)}\le\varepsilon\tag{6}$$

2. Analysis of the velocities of active modular group RRR-T$_a$-RRR

The system of position equations (2), with respect to the time, is derived and the following system is obtained [5-9;12]:

$$\begin{cases}-AB\sin\varphi_1\dot\varphi_1-BC\sin\varphi_2\dot\varphi_2+CD\sin\varphi_3\dot\varphi_3=A1\\AB\cos\varphi_1\dot\varphi_1+BC\cos\varphi_2\dot\varphi_2-CD\cos\varphi_3\dot\varphi_3=A2\\-AE\sin(\varphi_1-\alpha_1)\dot\varphi_1-S\sin\varphi_4\dot\varphi_4+DF\sin(\varphi_3+\alpha_2)\dot\varphi_3=A3\\AE\cos(\varphi_1-\alpha_1)\dot\varphi_1+S\cos\varphi_4\dot\varphi_4+DF\cos(\varphi_3+\alpha_2)\dot\varphi_3=A4\end{cases},\tag{7}$$

where

$$A_1=XD-XA;\ A_2=YD-YA;\ A_3=k-S\cos\phi_4;\ A_4=h-S\sin\phi_4.\tag{8}$$

The system of equations (7) is linear and it has the unknowns $\dot\varphi_1,\dot\varphi_2,\dot\varphi_3,\dot\varphi_4$ so:

$$\|\dot\varphi_1,\dot\varphi_2,\dot\varphi_3,\dot\varphi_4\|^T=W^{-1}\|A_1\quad A_2\quad A_3\quad A_4\|^T\tag{9}$$

3. Analysis of the accelerations of active modular group RRR-T$_a$-RRR

Through derivation of the system of velocity equations (7), with respect to the time, the system of acceleration equations is obtained [5-9;12]:

$$\begin{cases} -AB\sin\varphi_1\ddot{\varphi}_1 - BC\sin\varphi_2\ddot{\varphi}_2 + CD\sin\varphi_3\ddot{\varphi}_3 = B1 \\ AB\cos\varphi_1\ddot{\varphi}_1 + BC\cos\varphi_2\ddot{\varphi}_2 - CD\cos\varphi_3\ddot{\varphi}_3 = B2 \\ -AE\sin(\varphi_1 - \alpha_1)\ddot{\varphi}_1 + DF\sin(\varphi_3 + \alpha_2)\ddot{\varphi}_3 - S\sin\varphi_4\ddot{\varphi}_4 = B3 \\ AE\cos(\varphi_1 - \alpha_1)\ddot{\varphi}_1 - DF\cos(\varphi_3 + \alpha_2)\ddot{\varphi}_3 + S\cos\varphi_4\ddot{\varphi}_4 = B4 \end{cases} \tag{10}$$

with the following unknwons $\ddot{\varphi}_1, \ddot{\varphi}_2, \ddot{\varphi}_3, \ddot{\varphi}_4$, where:

$$\begin{cases} B1 = AB\cos\varphi_1\dot{\varphi}_1^2 + BC\cos\varphi_2\dot{\varphi}_2^2 - CD\cos\varphi_3\dot{\varphi}_3^2 + \ddot{X}D - \ddot{X}A \\ B2 = AB\sin\varphi_1\dot{\varphi}_1^2 + BC\sin\varphi_2\dot{\varphi}_2^2 - CD\sin\varphi_3\dot{\varphi}_3^2 + \ddot{Y}D - \ddot{Y}A \\ B3 = AE\cos(\varphi_1 - \alpha_1) \cdot \dot{\varphi}_1^2 + (S\dot{\varphi}_4^2 - \ddot{S})\cos\varphi_4 + 2\dot{S}\dot{\varphi}_4 - DF\cos(\varphi_3 + \alpha_2)\dot{\varphi}_3^2 + \ddot{X}D - \ddot{X}A \\ B4 = AE\sin(\varphi_1 - \alpha_1) \cdot \dot{\varphi}_1^2 + (S\dot{\varphi}_4^2 - \ddot{S})\sin\varphi_4 - 2\dot{S}\dot{\varphi}_4 - DF\sin(\varphi_3 + \alpha_2)\dot{\varphi}_3^2 + \ddot{Y}D - \ddot{Y}A \end{cases} \tag{11}$$

The solution of the system of equations (10) is:

$$\|\ddot{\varphi}_1, \ddot{\varphi}_2, \ddot{\varphi}_3, \ddot{\varphi}_4\|^T = W^{-1}\|B_1 \quad B_2 \quad B_3 \quad B_4\|^T. \tag{12}$$

In Figure 2 the kinematic scheme of a crank with the motortetrad is presented.

Fig. 2. **Kinematic scheme of crank with motor tetrad**
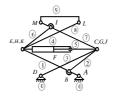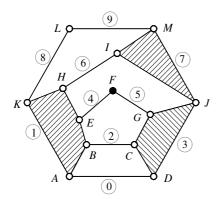
In Figure 3 the structural scheme corresponding to the mechanical system from Figure 2 is presented. The structural model has a binary element for the frame and it is composed by the RRR-T$_a$-RRR active modular group and two passive groups of RRR dyad type.
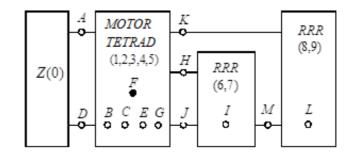

Fig. 3. **Structural scheme of crank with motor tetrad**

Fig. 4. **Multipolar scheme of crank with motor tetrad**

In Figure 4 the multipolar scheme corresponding to the structural scheme from Fig. 3 is presented. The actuating pair (F) is drawn as a black dot.

Results and discussions

To determine the kinematic parameters of the mechanism elements, the following data were considered: $AB = 0.061$ m, $AE = 0.280$ m, $BC = 0.1598$ m; $CD = 0.280$ m, $BE = AE-AB$, $JM = 0.280$ m, $JI = BE$, $HI = BC$, $KL = AE$, $ML = 0.180$ m, $XA = 0.0$ m, $YA = 0.0$ m, $XD = -0.180$ m, $YD = 0.0$ m, $S = 0.270$ m, stroke_hyd_cyl = 0.098 m, hyd_cyl speed $dCF/dt = -0.025$ m·s^{-1}, relative acceleration of the hydraulic cylinder = 0.0 m·s^{-2}

Using the procedure for the motor tetrad the kinematic parameters of the mechanical system presented in Figure 2 are calculated. The values for the angles between the axis OX and the vectors $\overline{AB}, \overline{BC}, \overline{DC}, \overline{EF}$ are presented in Table 3.

Table 3

Values of position parameters of the mechanical system

Pos.	0	1	2	3	4	5	6
φ_1	2.9266	2.8938	2.8640	2.8365	2.8108	2.7866	2.7635
φ_2	0.2698	0.3375	0.3936	0.4426	0.4866	0.5268	0.5641
φ_3	0.1999	0.2448	0.2823	0.3151	0.3444	0.3712	0.3958
φ_4	-0.0112	-0.0022	0.0036	0.0075	0.0103	0.0123	0.0136
Pos.	7	8	9	10	11	12	13
φ_1	2.7413	2.7200	2.6993	2.6792	2.6596	2.6404	2.6217
φ_2	0.5990	0.6319	0.6630	0.6927	0.7211	0.7483	0.7744
φ_3	0.4188	0.4404	0.4606	0.4799	0.4981	0.5155	0.5321
φ_4	0.0143	0.0145	0.0143	0.0136	0.0127	0.0113	0.0097
Pos.	14	15	16	17	18	19	20
φ_1	2.6032	2.5851	2.5672	2.5496	2.5321	2.5148	2.4977
φ_2	0.7997	0.8241	0.8477	0.8706	0.8928	0.9144	0.9355
φ_3	0.5480	0.5632	0.5779	0.5920	0.6055	0.6186	0.6312
φ_4	0.0077	0.0054	0.0028	-0.0001	-0.0032	-0.0067	-.00105

The values for the angular velocities of the links 1, 2, 3 and 4 (5) are presented in Table 4.

Table 4

Angular velocities of the mechanical system

Pos.	0	1	2	3	4	5	6
$\dot{\varphi}_1 = \omega 1$	-0.7043	-0.6363	-0.5825	-0.5411	-0.5087	-0.4826	-0.4613
$\dot{\varphi}_2 = \omega 2$	1.5520	1.2419	1.0633	0.9437	0.8563	0.7889	0.7348
$\dot{\varphi}_3 = \omega 3$	1.0240	0.8279	0.7106	0.6301	0.5703	0.5234	0.4853
$\dot{\varphi}_4 = \omega 4$	0.2370	0.1431	0.0966	0.0679	0.0477	0.0323	0.0198
Pos.	7	8	9	10	11	12	13
$\dot{\varphi}_1 = \omega 1$	-0.4435	-0.4286	-0.4158	-0.4047	-0.3952	-0.3868	-0.3795
$\dot{\varphi}_2 = \omega 2$	0.6901	0.6524	0.6199	0.5916	0.5666	0.5444	0.5243
$\dot{\varphi}_3 = \omega 3$	0.4535	0.4263	0.4027	0.3819	0.3633	0.3466	0.3315
$\dot{\varphi}_4 = \omega 4$	0.0091	-0.0002	-0.0086	-0.0163	-0.0236	-0.0305	-0.0371
Pos.	14	15	16	17	18	19	20
$\dot{\varphi}_1 = \omega 1$	-0.3731	-0.3674	-0.3624	-0.3580	-0.3541	-0.3507	-0.3477
$\dot{\varphi}_2 = \omega 2$	0.5061	0.4895	0.4743	0.4603	0.4473	0.4351	0.4238
$\dot{\varphi}_3 = \omega 3$	0.3176	0.3048	0.2930	0.2820	0.2717	0.2620	0.2529
$\dot{\varphi}_4 = \omega 4$	-0.0435	-0.0497	-0.0559	-0.0621	-0.0682	-0.0744	-0.0806

The values for the angular accelerations of the links 1, 2, 3 and 4 (5) are presented in Table 5.

Table 5

Angular accelerations of the mechanical system

Pos.	0	1	2	3	4	5	6
$\ddot{\varphi}_1 = \varepsilon 1$	1.4507	1.2506	0.9594	0.7449	0.5922	0.4813	0.3984
$\ddot{\varphi}_2 = \varepsilon 2$	-8.8132	-4.5881	-2.9137	-2.0584	-1.5539	-1.2275	-1.0020
$\ddot{\varphi}_3 = \varepsilon 3$	-5.4463	-2.9705	-1.9406	-1.3990	-1.0732	-0.8592	-0.7099
$\ddot{\varphi}_4 = \varepsilon 4$	-3.0928	-1.4133	-0.8644	-0.6199	-0.4901	-0.4131	-0.3639
Pos.	7	8	9	10	11	12	13
$\ddot{\varphi}_1 = \varepsilon 1$	0.3346	0.2845	0.2442	0.2113	0.1839	0.1608	0.1411
$\ddot{\varphi}_2 = \varepsilon 2$	-0.8386	-0.7156	-0.6204	-0.5448	-0.4836	-0.4332	-0.3911
$\ddot{\varphi}_3 = \varepsilon 3$	-0.6006	-0.5179	-0.4535	-0.4021	-0.3603	-0.3258	-0.2968
$\ddot{\varphi}_4 = \varepsilon 4$	-0.3309	-0.3080	-0.2919	-0.2806	-0.2727	-0.2674	0.2642
Pos.	14	15	16	17	18	19	20
$\ddot{\varphi}_1 = \varepsilon 1$	0.1241	0.1093	0.0962	0.0845	0.0741	0.0646	0.0559
$\ddot{\varphi}_2 = \varepsilon 2$	-0.3555	-0.3250	-0.2986	-0.2756	-0.2554	-0.2374	-0.2214
$\ddot{\varphi}_3 = \varepsilon 3$	-0.2723	-0.2513	-0.2331	-0.2173	-0.2033	-0.1910	-0.1800
$\ddot{\varphi}_4 = \varepsilon 4$	-0.2627	-0.2626	-0.2637	-0.2659	-0.2692	-0.2733	-0.2784

The variation of the kinematic parameters of the mechanical system is presented in Figure 5.
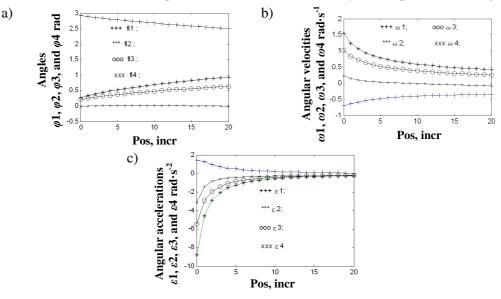

Fig. 5. **Variations of kinematic parameters:** a – positions; b – angular velocities;
c – angular accelerations

Conclusions

The paper presents a procedure to calculate the kinematic parameters (positions, angular velocities and accelerations) for the active modular group named motor tetrad RRR-T_a-RRR using the closed – loops method. Then, the kinematic analysis of a crank actuated by motor tetrad is presented. For the crank the structural and multipolar schemes and values of the kinematic parameters are presented.

The active modular group RRR-T_a-RRR can be used to actuate some mechanical systems, such as industrial machines and mechanisms, agricultural machinery etc.

The kinematic analysis of the mechanism in Figure 2 was done with the data obtained from the synthesis of the mechanism, so that the points M and N would move vertically for a certain distance required.

The kinematic analysis of the mechanism precedes its kinetostatic analysis, namely, determination of the reaction forces in the kinematic couplings and the driving force of the hydraulic cylinder, which will be done in the next research.

Acknowledgement

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